Equivalent Circuit of a Narrow-Wall Waveguide Slot Coupler

V. M. PANDHARIPANDE AND B. N. DAS

Abstract—**An expression for the equivalent network of a long axial slot in the common narrow wall between two rectangular waveguides is determined from self-reaction and discontinuity in modal current. The analysis is extended to the case of *H*-plane Tee junction coupled through a longitudinal slot in the narrow wall of primary guide. Variation of coupling, input VSWR, and impedance loading on the primary guide are determined from the equivalent network parameter. A comparison between theoretical and experimental results is presented.**

I. Introduction

THE coupling between two rectangular waveguides through apertures in the common broad wall has been analyzed by quasi-static antenna method [1], variational method [2], and from the equivalent circuit approach [3], [4]. A shortcoming of quasi-static antenna method by Lewin [1] is its inability to obtain from analysis, the accurate slot wave impedance to be used. The results are of limited application owing to a simplification which is made to permit the use of a well-known result of antenna theory. Formulas for the equivalent circuit parameter presented by Levy [4] and Marcuvitz [5] are applicable to small apertures. The analysis can, however, be extended to the case of large apertures by using Cohn's experimental results on polarizabilities [6], [7]. Levinson and Fredberg [8] presented a method for the determination of equivalent circuit of an aperture in the form of a long narrow transverse slot in the common broad wall. The formulation involves the solution of an integral equation and does not lead to a closed-form expression for the equivalent circuit parameter. They presented numerical results [9] for the equivalent circuit parameter for waveguides coupled through a narrow transverse slot from which coupling can be evaluated.

In the present paper, the equivalent circuit parameter for a long axial shunt slot in the common narrow wall is determined from self-reaction [10] and discontinuity in modal current [11]. Coupling and input VSWR are determined from the even- and odd-mode analysis. The method is also used for the determination of equivalent circuit of *H*-plane Tee junction coupled through a longitudinal slot in the narrow wall of primary guide and in the transverse cross section of the secondary guide. Variation of coupling, input VSWR, and impedance loading on the primary guide as a function of slot length are determined. Comparison between theoretical and experimental results is presented.

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II. Analytical Expression for the Shunt Reactance Due to the Longitudinal Slot Coupling Adjacent Guides

Fig. 1 shows two rectangular waveguides coupled through a longitudinal slot of length $2L$ and width d in the common narrow wall between two rectangular waveguides. The slot is located at a distance x_0 from the bottom broad wall of waveguides. The electric field distribution $\bar{E}_s$ in the aperture plane of the thin longitudinal slot can be considered equivalent to a surface magnetic-current distribution $\bar{J}_z^m$, and the two are related by

$$\bar{J}_z^m = \bar{E}_s \times \bar{n} \tag{1}$$

where $\bar{n}$ is the unit vector normal to the aperture plane. The aperture field $\bar{E}_s$ is assumed to be of the form

$$\bar{E}_s = \bar{u}_x E_0 \sin k(L-|z|)\delta(x'-x_0)\delta(y'-a) \tag{2}$$

x_0, a, 0 are the coordinates of the center of the slot, E_0 is the maximum electric field in the slot, and $k=2\pi/\lambda$.

The vector potential due to longitudinal magnetic-current density $\bar{J}_z^M$ is given by [12]

$$A_z = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{\epsilon_n \epsilon_m}{2ab\gamma}\cos\left(\frac{n\pi x}{b}\right)\cos\left(\frac{m\pi y}{a}\right) \cdot \int_s \cos\left(\frac{n\pi x'}{b}\right)\cos\left(\frac{m\pi y'}{a}\right) \cdot\left[e^{-\gamma z}\int_{-\infty}^{z} \bar{J}_z^M e^{\gamma z'}dz' + e^{\gamma z}\int_{z}^{\infty}\bar{J}_z^M e^{-\gamma z'}dz'\right]dx'dy' \tag{3}$$

where

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2}$$

and ϵ_n and ϵ_m are Neumann numbers. For the slot extending from $-L$ to $+L$, performing the surface integration over the aperture, the expression for vector potential using (1) and (2) reduces to

$$A_z = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{-\epsilon_n\epsilon_m V_m}{ab\gamma}\cos\left(\frac{n\pi x}{b}\right)\cos\left(\frac{m\pi y}{a}\right) \cdot \cos m\pi \cos\left(\frac{n\pi x_0}{b}\right)\left\{\frac{\sin\left(\frac{n\pi d}{2b}\right)}{\left(\frac{n\pi d}{2b}\right)}\right\}\cdot\frac{k}{k^2+\gamma^2} \cdot\left[e^{-\gamma|z|}\cos kL - e^{-\gamma L}\cosh\gamma z - \frac{\gamma}{k}\sin k(L-|z|)\right] \tag{4}$$

where $V_m = E_0 \cdot d$.

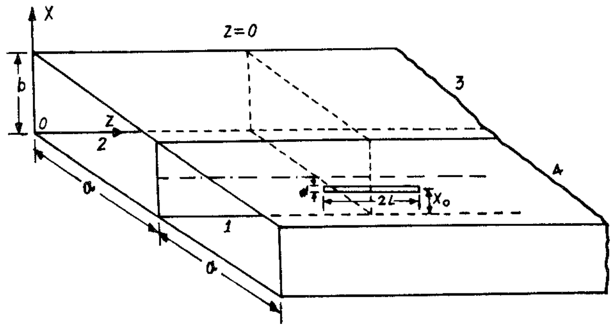

Fig. 1. Rectangular waveguides coupled through longitudinal slot in the common narrow wall.

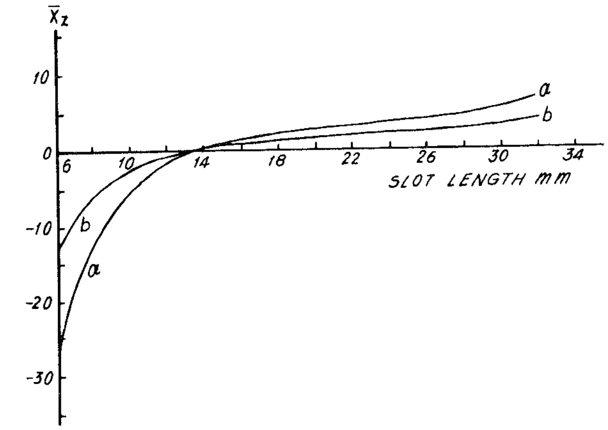

Fig. 2. Variation of normalized reactance versus slot length for different displacements from center. $\lambda=3.2$ cm; Curve a—$(b/2-x_0)=2$ mm; Curve b—$(b/2-x_0)=4$ mm.

The magnetic field H_z is obtained from the vector potential using Maxwell's equation as

$$H_z=\frac{1}{j\omega\mu}\left[k^2+\frac{\partial^2}{\partial z^2}\right]A_z. \tag{5}$$

The self-reaction $\langle a,a\rangle$ of magnetic current $\bar{J}_z^M$ is given by

$$\langle a,a\rangle=-\int_v \bar{H}_z\cdot\bar{J}_z^M dv. \tag{6}$$

A part of coupled volume v is in the primary guide and the other part is in the secondary (coupled) guide. In the general case of two dissimilar guides, the total self-reaction is equal to the sum of self-reactions $\langle a,a\rangle_1$ and $\langle a,a\rangle_2$ in the two volumes. In the particular case of two identical guides, the total self-reaction is equal to twice that of a single guide.

Using (1), (2), (4)–(6) and evaluating the integrals, the total self-reaction of the equivalent magnetic current of a longitudinal slot in the common narrow wall of two identical waveguides reduces to

$$\langle a,a\rangle=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{4j\epsilon_n\epsilon_m V_m^2 k^2\cos^2 m\pi}{\omega\mu ab\gamma(k^2+\gamma^2)}\cdot\cos^2\left(\frac{n\pi x_0}{b}\right)\left\{\frac{\sin^2\frac{n\pi d}{2b}}{\left(\frac{n\pi d}{2b}\right)^2}\right\}\cdot\left[0\cdot 5(1+e^{-2\gamma L})-\cos kL\left(2e^{-\gamma L}\cos kL+\frac{\gamma}{k}\sin kL\right)\right]. \tag{7}$$

A longitudinal slot in the waveguide wall produces a discontinuity in the modal current, giving rise to shunt type of equivalent network parameter $B_z=1/x_z$, where x_z is given by [10], [13]

$$x_z=\frac{\langle a,a\rangle}{I^2}. \tag{8}$$

The discontinuity in the modal current I for the dominant mode is obtained from the expression [11]

$$I=jY_{01}\int_{\text{slot}}\bar{n}\times\bar{E}_s\left(\bar{h}\sin\beta_{01}z+j\bar{h}_z\cos\beta_{01}z\right)ds \tag{9}$$

where

$$\bar{h}=\bar{u}_y\frac{\pi}{a}\sin\frac{\pi y}{a},\qquad \bar{h}_z=\bar{u}_z\frac{(\pi/a)}{j\beta_{01}}\cos\frac{\pi y}{a},$$

$$\beta_{01}=\sqrt{k^2-(\pi/a)^2}.$$

Evaluating the integral appearing in (9) over the slot aperture and using (7) and (8), the expression for the normalized reactance is obtained in the form

$$\bar{x}_z=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{-\epsilon_n\epsilon_m\beta_{01}\cos^2 m\pi}{ab\gamma(k^2+\gamma^2)}\left\{\frac{\cos\frac{n\pi x_0}{b}}{\cos\frac{\pi x_0}{b}}\right\}^2\left\{\frac{\sin\frac{n\pi d}{2b}}{\frac{n\pi d}{2b}}\right\}^2\cdot\left[\frac{0.5(1+e^{-2\gamma L})-\cos kL\left(2e^{-\gamma L}-\cos kL+\frac{\gamma}{k}\sin kL\right)}{(\cos\beta_{01}L-\cos kL)^2}\right]. \tag{10}$$

The summation is carried out for all modes excepting $m=n=0$ and $n=0$, $m=1$.

Fig. 2 shows the variation of normalized reactance as a function of slot length computed for $\lambda=3.2$ cm, slot width $d=1$ mm, waveguide dimensions $a=2.286$ cm, $b=1.016$ cm, displacement from center $((b/2)-x_0)=2$ and 4 mm. Computation of the double infinite series is carried out considering terms up to $m=7$ and $n=7$, thus taking the effect of 62 evanascent modes into account. The value of reactance for a particular slot length decreases with increasing displacement from the center of the narrow wall $(x_0=b/2)$, showing resonance at slot length $2L=1.42$ cm.

III. Coupling Due to Shunt Slot in Terms of Equivalent Network Parameter

Fig. 3(a) shows the lumped parameter equivalent network for the coupled waveguides looking from port 1 of Fig. 1. In the equivalent transmission-line representation, Fig. 3(a) assumes the form shown in Fig. 3(b). The corresponding even- and odd-mode [4] equivalent circuits are shown in Fig. 3(c).

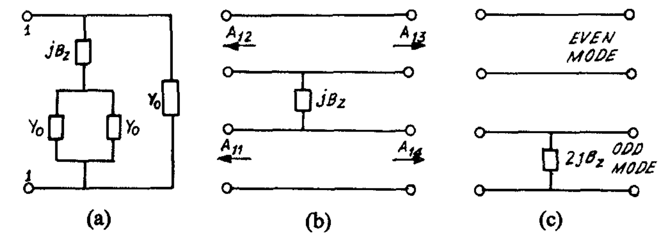

Fig. 3. Equivalent circuit of coupled waveguides.

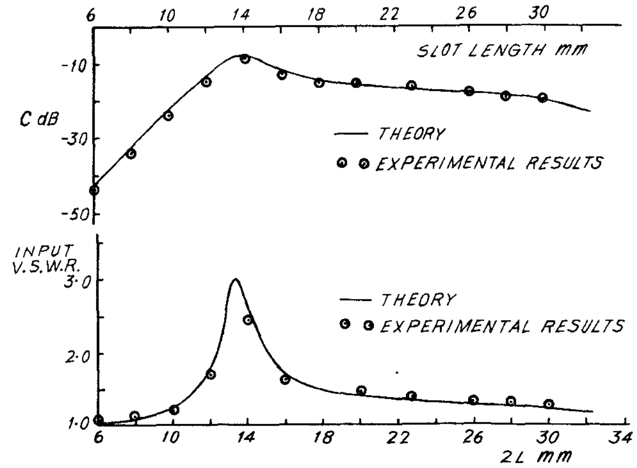

Fig. 4. Variation of coupling and input VSWR versus slot length. $\lambda = 3.2$ cm; displacement from center $(b/2 - x_0) = 2$ mm; slot width 1 mm.

Analyzing the network using even- and odd-mode equivalent circuit of Fig. 3(c), it is found that the magnitude of reflection coefficient $|A_{11}|$ at port 1 and coupling coefficients $|A_{12}|$ and $|A_{13}|$ at ports 2 and 3 are identical, and are given by [4]

$$|A_{11}| = |A_{12}| = |A_{13}| = \left| \frac{j\bar{B}_z}{2(1 + j\bar{B}_z)} \right| \tag{11}$$

where $\bar{B}_z = 1/\bar{x}_z$. The same expression can also be obtained by considering the equivalent circuit of Fig. 3(a).

The variational expression for the equivalent network parameter obtained above is based on the assumption that the slot walls are of zero thickness. The effect of finite wall thickness t can be taken into account by applying correction due to the attenuation α in a waveguide whose cross section is the same as that of the slot and whose length is equal to the wall thickness t. Hence, applying the wall-thickness correction, coupling in decibels is given by

$$A_{13} = 20 \log \frac{1/\bar{x}_z}{2\sqrt{1 + 1/\bar{x}_z^2}} - 8.686\alpha t \tag{12}$$

$$\text{Input VSWR} = \frac{1 + |A_{11}|}{1 - |A_{11}|}. \tag{13}$$

Variation of coupling and input VSWR with slot length is computed for a slot of width 1 mm and displaced 2 mm from the plane, passing through the axis of the waveguide and normal to the narrow wall. For the same slot located in a common narrow wall of wall thickness $t = 1.5$ mm, coupling and input VSWR was measured experimentally. Comparison of results computed from (12) and (13) with the experimental results is shown in Fig. 4 for slot length over the range 8 mm $< 2L <$ 30 mm. A good agreement is observed between theory and experiment.

IV. Analysis of Slot Coupled Tee Junction in Rectangular Guide H-Plane

The same analysis can be extended to evaluate the coupling for the case of H-plane Tee junction coupled through longitudinal slot in the narrow wall of primary guide and in the transverse cross section the secondary guide as shown in Fig. 5(a). The corresponding lumped parameter equivalent network looking from port 1 of primary guide is shown in Fig. 5(b). The equivalent network parameter is again given by the expression of the form (8). Self-reaction $\langle a,a \rangle$ in this case is determined separately for the two guides. The self-reaction $\langle a,a \rangle_1$ in guide I is $1/2\langle a,a \rangle$, where $\langle a,a \rangle$ is given by (7). The self-reaction $\langle a,a \rangle_2$ in secondary guide (guide II), obtained from the modal expansion of the magnetic field in this guide, is given by [14]

$$\langle a,a \rangle_2 = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{4jk^2 \epsilon_n V_m^2}{\omega\mu a b \gamma} \sin^2\left(\frac{m\pi}{2}\right) \cos^2\left(\frac{n\pi x_0}{b}\right) \cdot \left\{ \frac{\sin \frac{n\pi d}{2b}}{\frac{n\pi d}{2b}} \right\}^2 \cdot \left\{ \frac{\left(\cos \frac{m\pi L}{a} - \cos kL\right)^2}{k^2 - \left(\frac{m\pi}{a}\right)^2} \right\}. \tag{14}$$

Hence, the equivalent network parameter will be

$$X_{zT} = \frac{\langle a,a \rangle_1 + \langle a,a \rangle_2}{I^2}. \tag{15}$$

From (7), (9), (14), and (15) the expression for normalized reactance for a slot coupled Tee junction reduces to

$$\bar{X}_{zT} = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{\beta_{01}}{2ab\gamma} \cdot \left\{ \frac{\cos^2 \frac{n\pi x_0}{b}}{\cos^2 \frac{\pi x_0}{b}} \right\} \left\{ \frac{\sin \frac{n\pi d}{2b}}{\frac{n\pi d}{2b}} \right\}^2 \left[\frac{\epsilon_n \epsilon_m}{k^2 + \gamma^2} \cdot \left\{ 0.5(1 + e^{-2\gamma L}) - \cos kL\left(2e^{-\gamma L} - \cos kL + \frac{\gamma}{k} \sin kL\right) \right\} - 2\epsilon_n \sin^2 \frac{m\pi}{2} \frac{\left(\cos \frac{m\pi L}{a} - \cos kL\right)^2}{\left(\frac{m\pi}{a}\right)^2 - k^2} \right] \Big/ (\cos \beta_{01} L - \cos kL)^2. \tag{16}$$

From the equivalent circuit shown in Fig. 4(b) the expression for the input reflection coefficient, and hence

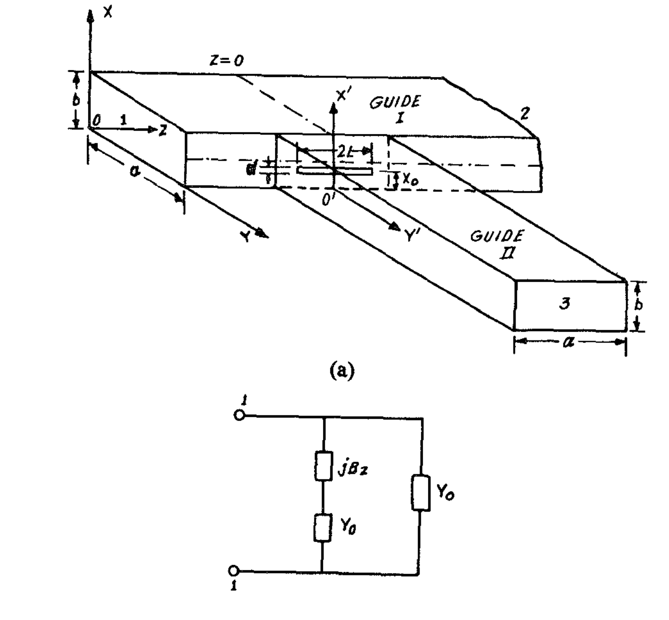

Fig. 5(a) H-plane Tee junction coupled through longitudinal slot in the primary guide. (b) Equivalent circuit.

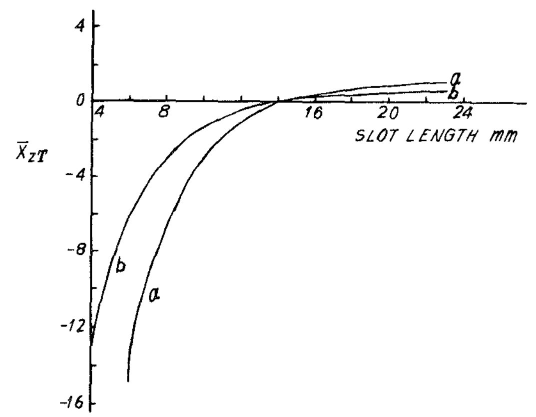

Fig. 6. Variation of normalized reactance versus slot length for H-plane Tee junction. $\lambda=3.2$ cm; Curve a—displacement from center $(b/2-x_0)=2$ mm; Curve b—displacement from center $(b/2-x_0)=4$ mm.

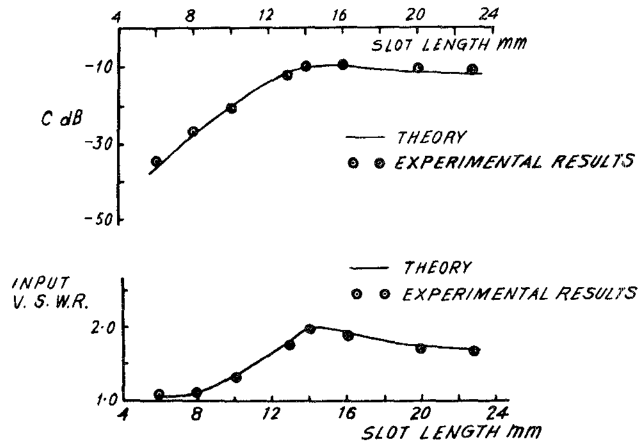

Fig. 7. Variation of coupling and input VSWR versus slot length $\lambda=3.2$ cm; slot width $=1$ mm; displacement from center $(b/2-x_0)=2$ mm.

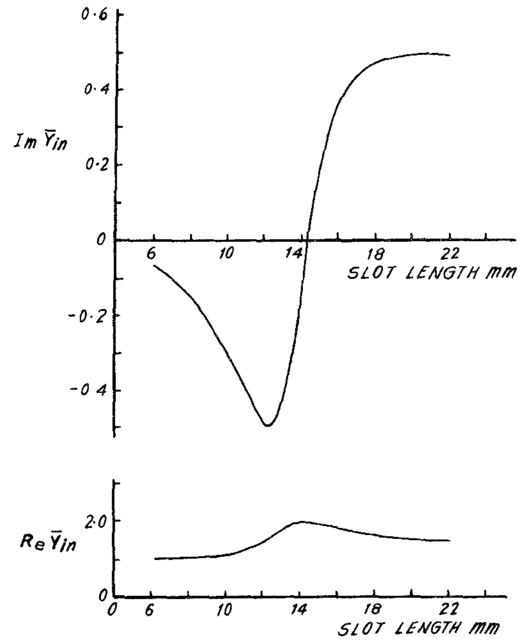

Fig. 8. Variation of real and imaginary parts of input admittance versus slot length, for $\lambda=3.2$ cm.

the coupling coefficient is given by [14]

$$A_{11}=A_{13}=\left|\frac{j\bar{B}_{zT}}{2+3j\bar{B}_{zT}}\right| \tag{17}$$

where $\bar{B}_{zT}=1/\bar{X}_{zT}$.

For a finite wall thickness t, coupling in decibels can be written as

$$A_{13}=20\log\frac{1/\bar{X}_{zT}}{\sqrt{4+9/\bar{X}_{zT}^2}}-8.686\alpha t \tag{18}$$

$$\text{Input VSWR}=\frac{1+|A_{11}|}{1-|A_{11}|}. \tag{19}$$

Fig. 6 shows the variation of normalized reactance with slot length computed from (16), for $\lambda=3.2$ cm, $a=2.286$ cm, $b=1.016$ cm, and slot width $d=1$ mm for two displacements of slot (2 mm and 4 mm from center). In order to determine coupling experimentally, the incident power is first measured by removing the Tee junction and terminating the guide in a matched load. The Tee junction is then inserted between the matched load and the guide. With the port 3 terminated in a matched load, the input VSWR at port 1 and power in port 3 are measured. Fig. 7 shows the comparison of experimental results with those computed from (18) and (19).

V. Impedance Loading on the Primary Guide

For the design of cascaded sections of such Tee junctions using loaded line analysis [15], a knowledge of the impedance loading on the primary guide is essential.

From the equivalent circuit of Fig. 4(b), we can write the expression for normalized admittance, presented to guide I, in terms of equivalent network parameter as follows:

$$\bar{Y}_{\text{in}} = \left(1 + \frac{1}{1+\bar{X}_{zT}^2}\right) + \frac{j\bar{X}_{zT}}{1+\bar{X}_{zT}^2}. \tag{20}$$

Fig. 8 shows the variation of real and imaginary parts of input admittance as a function of slot length computed from (20) using (16) for $\lambda = 3.2$ cm and slot displacement from center = 2 mm.

VI. Conclusion

Theoretical results for coupling, input VSWR are in good agreement with the experimental results for the two adjacent waveguides coupled by means of a long narrow slot in the common narrow wall, as well as for the case of H-plane Tee junction. The present theory leading to a closed-form expression for the equivalent circuit parameter for coupling apertures is quite general and can be applied for the analysis of electromagnetic coupling between any two arbitrary guides.

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Numerical Calculation of Electromagnetic Energy Deposition for a Realistic Model of Man

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***Abstract*—Numerical calculations of absorbed energy deposition have been made for a block model of man that is defined with careful attention given to the biometric and anatomical features of a human being. Calculated post-resonant absorption and distribution of energy deposition through the body have better agreement with experimental results than previous calculations made using less realistic models.**

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I. Introduction

WE HAVE used numerical methods with a realistic model to calculate the deposition of electromagnetic energy in man. A block model of man was chosen to allow maximum freedom in defining both the shape and content of the model. We have used the moment-method solution of the electric-field integral equation with a pulse function basis and delta functions for testing [1], [2]. The

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